

Original Article

# Active Vibration Control of a Flexible Spacecraft Structure

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In this paper, the spacecraft that evaluated has two motion mode, rigid body mode and flexible mode and it is operated in low earth orbit. The rigid body mode is related to the attitude of spacecraft and flexible mode is related to vibration that occurs on the spacecraft structure. The vibration that occurs on the spacecraft structure may cause performance degradation during operation. Hence, the active control vibration is applied to overcome the problem due to the vibration phenomenon on spacecraft. The active control system was designed by using two methods, Pole-Placement Method and Linear Quadratic Regulator (LQR) Method, and those two methods are solved by using numerical method. The result of Pole-Placement Method shows the vibration is reduce in less than 0.5 unit of time. Whereas, the most suitable control parameter input based on the LQR Method could reduce vibration in less than 8 unit of time. The LQR method provides more parameter variation; thus, the system could be controlled and adjusted due to its design requirement. Based on the LQR Method when the attenuation time is 8 unit of time, the energy required by the actuator is 84% less than that of the Pole-Placement Method.

**Keywords:** Vibration, Active Control System, Spacecrafts

## 1. Introduction

The use of spacecraft for research and development of science, conducting surveillance, and supporting needs such as communication and navigation on earth requires that spacecraft, especially satellites that have good performance. One of part the satellite that has an important role in maintaining performance is the solar panel, where in this study the type of deployable solar panel structure was studied. The function of solar panels as a source of power for satellites during orbiting the earth needs special attention.

In this research, the spacecraft that studied is satellite which is orbiting in low earth orbit. Disturbances that may occur in these orbital conditions are caused by several things such as the earth's gravitational field, micrometeorite rain, and solar radiation. Among these disturbance, the most dominant disturbance is the gravitationsl force gradient, so that along the solar panel it will affect the load on the structure borne. Meanwhile, the disturbance of micrometeorite rain causes random disturbances and is a little difficult to model and interference from solar radiation will only cause thermal distortion. Both of these disturbances will only give a disturbance that has a small magnitude [1]. The model of a large spacecraft structure is represented through several coordinate reference systems, and is defined into two parts, namely the rigid object and the flexible object [2].

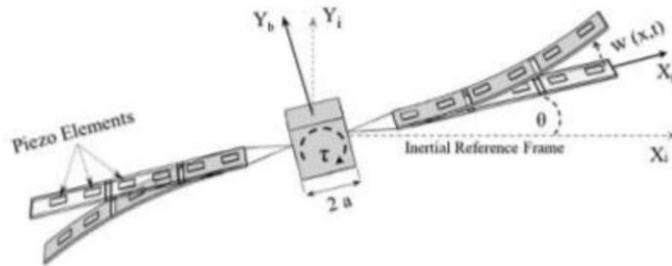


Figure 1. Illustration of Spacecraft with Flexible Solar Panel Structure [3]

The load that experienced by the spacecraft’s solar panel structure is exciting the structure and lead to vibration of the structure. There are two methods that can be conducted to reduce the vibration that occurs in the structure and increasing the rigidity of the solar panel structure. However, when increasing the rigidity of the structure, the weight of the vehicle will increase and will affect the cost of launching it [4]. The study of structural load due to gravitational force gradients is also discussed in one of the literature references, where the application of control and optimization of the structure that can be reduced weight as much as 20% when the control is applied without even violating the limits of damping factors and vibration control in the structure [5].

The control system was designed to reduce vibration on structure of spacecraft were aimed for controlling the attitude of spacecraft. Due to the dynamic coupling between rigid body mode and flexible mode, such as in The Hubble Telescope, the damping characteristics was increased from 0.5% to 2.3% [6]. Nowadays, the development of smart structure material such as piezoelectric is used to control the vibration of spacecraft structure and shows significant performance improvement when compared to contributions of the attitude control to the vibration damping [7].

## 2. Flexible Spacecraft Dynamics

Simply, the spacecraft structure is assumed as beam model that only can move in the bending motion. In order to model and describe the physical matters of the spacecraft, the coordinate reference system is used to express the spacecraft in its orbit.

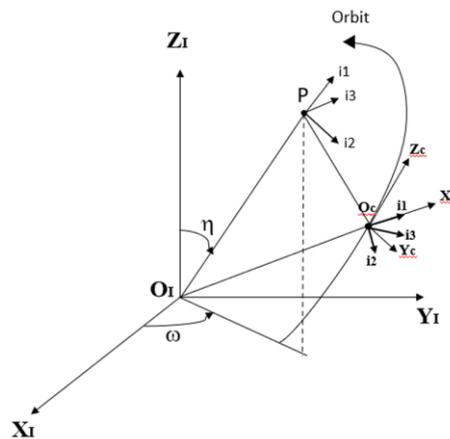


Figure 2. Integrated Coordinate Reference System

For transforming each coordinate reference system to another, the transformation matrix is derived to explain it mathematically. There are some transformation matrix that derived and used to comply it.

$$T_1 = \begin{bmatrix} s\eta c\omega & s\eta s\omega & c\eta \\ c\eta c\omega & c\eta s\omega & -s\eta \\ -s\omega & c\omega & 0 \end{bmatrix} \quad (2.1)$$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cx & sx \\ 0 & -sx & cx \end{bmatrix} \quad (2.2)$$

$$T_3 = \begin{bmatrix} c\phi c\theta & s\phi c\psi + c\phi s\theta s\psi & s\phi s\psi - c\phi s\theta c\psi \\ -s\phi c\theta & c\phi c\psi - s\phi s\theta s\psi & c\phi s\psi + s\phi s\theta c\psi \\ s\theta & -c\theta s\psi & c\theta c\psi \end{bmatrix} \quad (2.3)$$

By definition, T1 matrix is transforming the vector on P reference into its inertial reference, T2 matrix is transforming the vector on local reference into its orbital reference, and T3 matrix is transforming the orbital reference into its body reference.

The spacecraft has two type of motion which are rigid body mode and flexible mode. The rigid body mode is related to the attitude of the spacecraft and the flexible mode is related to the vibration that occurs on the structure. By using the 2nd Newton's Law and assumed that the orbit is a perfect circle, the equation of motion of rigid body mode was obtained and expressed in this following equation.

$$\ddot{\theta} + \dot{\Omega}_{oy} + \frac{C_y}{J_y} + \frac{3}{2} \omega_c^2 \sin 2\theta = 0 \quad (2.4)$$

where

$\theta$  – pitch angle of spacecraft

$\omega_c$  – orbital angular speed

$\dot{\Omega}_{oy}$  – spacecraft body angular speed

$C_y$  – torque which is result from external disturbance that act on the spacecraft body in terms of y-axis

$J_y$  – Spacecraft's moment of inertia (undeformed) in terms of y-axis

Due to the definition of mode shape and assumed that the displacement is only occurs in the z direction in terms of body reference, the flexible mode of the spacecraft is expressed in this following equation.

$$\frac{d^2 Z_n}{d\tau^2} + (\delta + \varepsilon \cos(2\tau) + \eta) Z_n = 0 \quad (2.5)$$

where

$$\delta = \frac{4}{3} \left\{ \left( \frac{\omega_n}{\omega_c} \right)^2 - \frac{3c^2}{2} \right\}, \varepsilon = \frac{8\sqrt{3}}{3} C - \frac{8\Omega_{oy}}{3\omega_c} \sqrt{3} C, \text{ and } \eta = 2 \frac{\Omega_{oy}}{\omega_c}$$

By Definition

$Z_n$  – Amplitude of vibration on a point that being observed divided by structural length (non-dimensional)

$C$  – Amplitude of pitch

$\omega_n$  – structural natural frequency

Specifically, the dynamic characteristics of flexible mode was obtained from the result of vibration analysis which was conducted by NASA researcher on their spacecrafts [8], the choosen natural frequency is 0.08 Hz with the in plane bending vibration mode.

### 3. Implementation of Active Control Vibration

The vibration of spacecraft structure is overcome with active control vibration system. Conventionally, the active control vibration system on this spacecraft structure has a mathematical form as this following equation.

$$\frac{d^2Z_n}{d\tau^2} + (\delta + \varepsilon \cos(2\tau) + \eta)Z_n = F_{control} \quad (3.1)$$

where, the  $F_{control}$  is the control force that reduce the vibration. In this research, there are two methods that used to obtain  $F_{control}$  or model the active control vibration system mathematically, The Pole-Placement Method and Linear Quadratic Regulator (LQR) Method. The Pole-Placement Method are applied by giving force which related to velocity parameter on the structure directly [9]. Mathematically by using The Pole-Placement Method, the equation of flexible mode motion is expressed in this following.

$$F_{control} = -F \frac{dZ_n}{d\tau} \quad (3.2)$$

where F is the control gain of Pole-Placement Method which is related to velocity of vibration. The amount value of F is determined at the limit of its characteristic equation, the limitation of F is expressed in this following mathematical express.

$$F = \begin{cases} \zeta \sqrt{(\delta + \varepsilon + \eta)}, & \cos(2\tau) = 1 \\ \zeta \sqrt{(\delta - \varepsilon + \eta)}, & \cos(2\tau) = -1 \end{cases} \quad (3.3)$$

By tuning the damping force coefficient ( $\zeta$ ), the characteristic roots of system is located on left side of the imaginary axis.

Another method that used in the implementation os active control vibration is The Linear Quadratic Regulator (LQR) Method. The control force that developed by using Linear Quadratic Regulator (LQR) Method can be expressed as this following equation

$$F_{control} = -K_1 Z_n - K_2 \frac{dZ_n}{d\tau} \quad (3.4)$$

where K1 and K2 are the control gain that are related to displacement and velocity of vibration respectively. This method is derived by obtaining the solution of the cost function (J)

$$J = \int_0^{\infty} (\bar{x}^T Q \bar{x} + u^T R u) dt \quad (3.5)$$

Where  $Q$  is related to the system performance and  $R$  is related to the input that given to the system [10]. The solution is obtained by solving the Algebraic Riccati Equation (ARE) which is obtained by defining the control signals that input to the system with this following equation

$$u = -Kx \tag{3.6}$$

Schematically, the sensor of vibration is accelerometer that capture acceleration, then derived it once for gain of the Pole-Placement Method and derived it twice for gain of LQR Method since the control gain also related to the displacement. By comparing these two methods, the control forces that giving the most efficient performance is being observed and analyzed in order to determine which is the most suitable method on this research.

#### 4. Result and Discussion

The vibration that occurs on the spacecraft structure is evaluated early to observe the vibration characteristic. By vary the parameters, such as pitch angle, spacecraft orbital angular speed, and spacecraft body angular speed, the vibration characteristics can be known. One of the most interesting parameters is the pitch angle, the structural response due to this parameter is shown in this following figure.

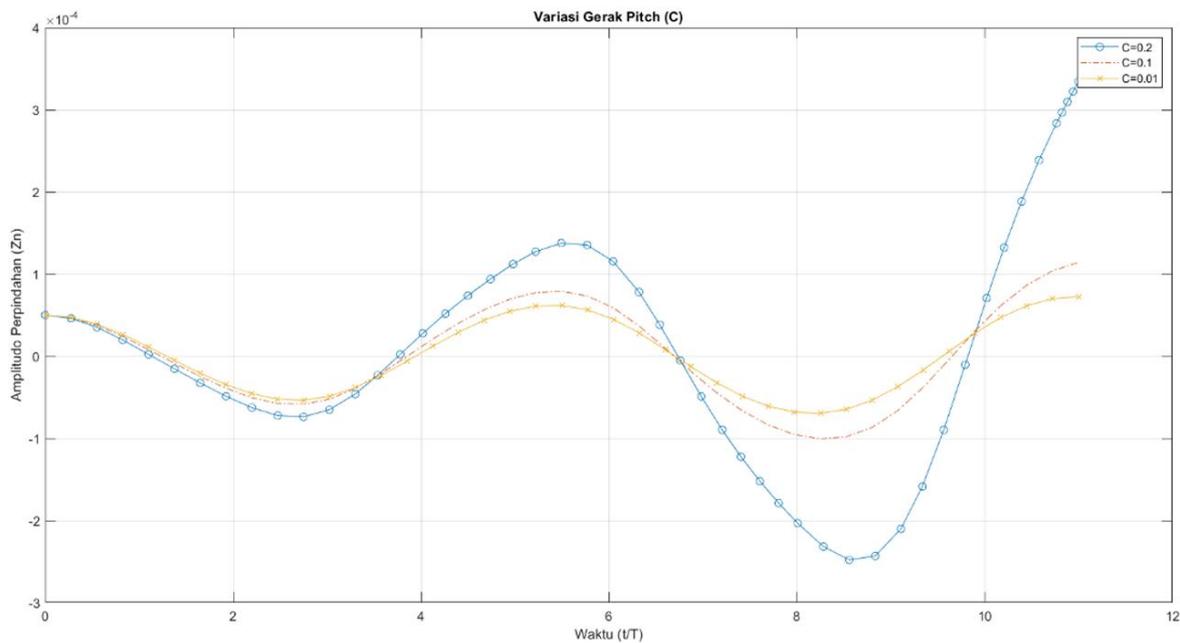
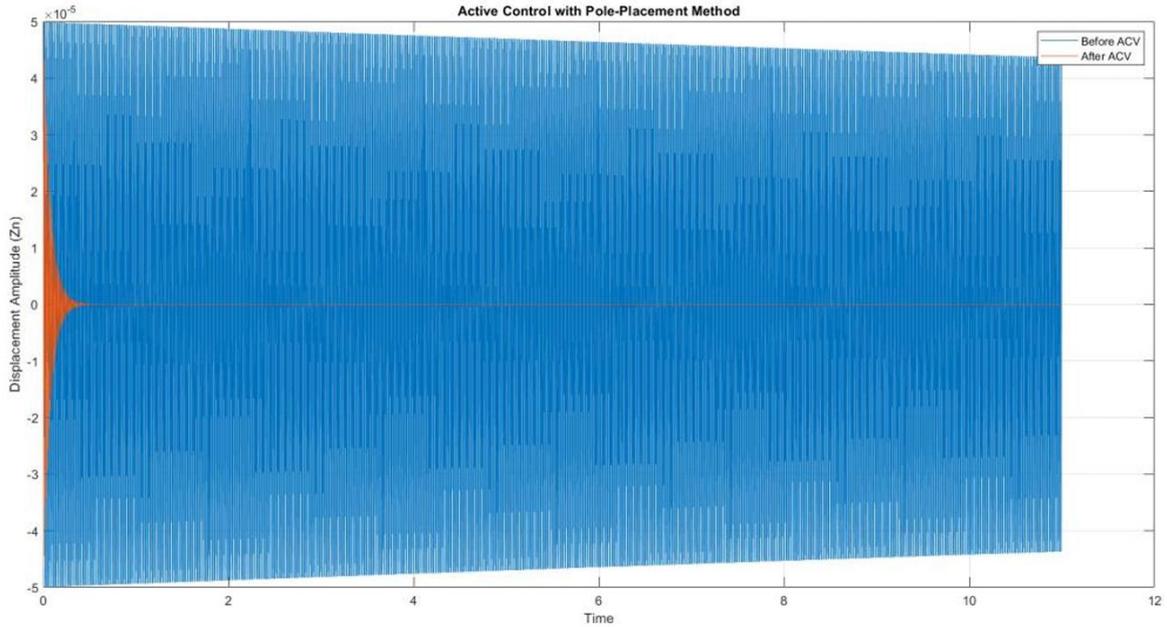


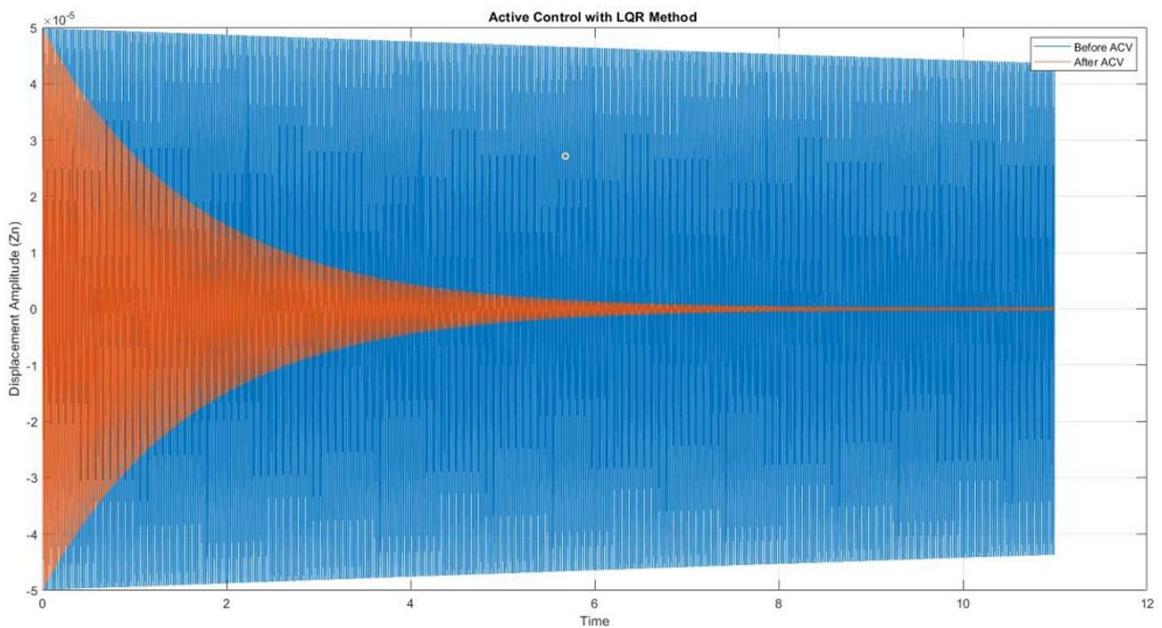
Figure 3. Structural Response due to Variation of Spacecraft Pitch Angle

The characteristics of vibration with a certain pitch value cause unstable or divergence oscillation and this is also prove that there is dynamic coupling between rigid body mode and flexible mode of spacecraft.



**Figure 4.** Implementation of Active Control Vibration on Spacecraft Structure with Pole-Placement Method

The active control vibration with The Linear Quadratic Regulator Method is also conduct by tuning The Q and R value based on equation (3.3), the different value of Q and R is resulted different K1 and K2 which is the control gain that act as control forces to reduce the vibration. One of the most efficient solution or suitable value of Q and R is giving that the value of K1 is 2.59E-06 and K2 is 1.183. Hence, the structural response is obtained and shown in this following figure.



**Figure 5.** Implementation of Active Control Vibration on Spacecraft Structure with Linear Quadratic Regulator Method

It can be seen that the active control with LQR Method can be better damped in a not too short attenuation time. For comparing between those two methods more intuitively, another parameter

that calculated is the energy that needed to reduce the vibration. Simply the energy calculation is calculated by using this following equation

$$W = \int_0^t F dz \tag{4.1}$$

Where F is the amount of control forces that act on the system and it is integrated due to the change of vibration amplitude in z direction.

**Table 1.** Comparison of Linear Quadratic Regulator Methode and Pole-Placement Method on The Spacecraft Structure in General

No.	Linear Quadratic Regulator Method				Pole-Placement Method		
	K1	K2	Attenuation Time	Energy	F	Attenuation Time	Energy
1	7.41.E-07	0.632	16	0.0021	25.94	0.5	0.0327
2	1.85.E-06	1.000	10	0.0033			
3	2.59.E-06	1.183	8	0.0039			
4	3.70.E-06	1.414	7	0.0046			
5	1.85.E-06	1.000	10	0.0033			
6	1.54.E-06	0.913	11	0.003			
7	1.23.E-06	0.816	12	0.0027			
8	1.09.E-06	0.767	13	0.0025			
9	9.26.E-07	0.707	14	0.0023			
10	3.70.E-04	14.142	1	0.0327			
11	7.41.E-04	20.000	0.7	0.0462			
12	1.95.E-08	0.100	100	0.00032			
13	9.74.E-09	0.071	140	0.00023			
14	1.85.E-06	1.000	10	0.0033			

Based on the table above, it can be seen that The LQR method is resulting various attenuation time with less amount of energy to reduce the vibration than The Pole-Placement Method. So that the actuator that design with LQR Method does not have to work as much as Pole-Placement Method. The Pole-Placement Method is resulting higher energy to reduce the vibration due to the short attenuation time.

**5. Conclusion**

Based on the result that already obtained in this research, the vibration on that spacecraft structure is more effectively reduce by using the active control system which is designed with The Linear Quadratic Regulator (LQR) Method. The amount of control gain that used in the LQR Method can be adjusted due to the tuned value of Q and R. The amount of control gain by those two methods is compared generally in Table 1. It is shown that the Linear Quadratic Regulator (LQR) Method is required 84% less energy than the Pole-Placement Method.

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